Cascade Tuning for Nonlinear Position Control of an Electrohydraulic Actuator

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Abstract—The nonlinear position control of an electrohydraulic actuator is approached by using two control structures. The first is a direct near IO linearization of the system model with piston position as output. The second is a cascade controller with a near IO linearizing pressure force controller as an inner-loop to a feedback plus feed forward outer-loop position controller. It is shown in this paper that the two control structures are theoretically equivalent. Furthermore, the equivalence is exploited to extract a simple, physically intuitive, tuning procedure for the gains of the two controller structures. This is particularly significant for the near IO linearizing position controller whose gains lack a physically tractable interpretation that guides their selection.

Keywords: electrohydraulic actuator, cascade control, IO linearization, position control

I. INTRODUCTION

Electrohydraulic actuators are used in a variety of positioning and force generation applications where their high load stiffness and high power-to-weight ratio make them better choices than other rival actuation systems. However, electrohydraulic actuators exhibit significant nonlinearities in their dynamics. To obtain satisfactory performance in the presence of these nonlinearities, elaborate nonlinear controllers are often necessary.

The literature offers a wide variety of methods for improving the position and force tracking performance of electrohydraulic actuators. These include variants of linear state feedback [1], adaptive control [1-4], variable structure control [5] and Lyapunov-based controller designs [2, 6-8]. Each approach has its own strengths and limitations, which are outlined in the respective listed references.

In this paper, two versions of position controllers are derived based on feedback linearization. A formal theory of feedback linearization is detailed in the texts [9, 10]. Feedback linearization involves the transformation of a nonlinear system to a linear one via nonlinear state feedback and input transformation. The linear system can then be handled using results from linear control theory.

Perhaps the earliest study on the application of feedback

linearization to electrohydraulic actuators was that of Axelson and Kumar [11] in 1988. They presented the derivation of the control laws emphasizing the nonlinearity of valve flow only. Hahn, et al [12] derived a more detailed controller for the position tracking case and presented limited results from simulations with an inertia load. Vossoughi and Donath [13] presented an analysis and derivation of feedback linearizing controllers for velocity tracking in a robotics application.

In the strictest sense, some assumptions are necessary to put the model of an electrohydraulic actuator in a form that approaches a partial feedback linearizable or an input-output (IO) linearizable form. These necessary assumptions, often tacitly overlooked in prior work, will be explicitly listed in this paper. To make distinctions from a true IO linearization, we use the name near IO linearization.

The focus of this paper is to present a tuning procedure for the near IO linearizing position tracking controller by first showing its equivalence with a cascade controller. The cascade control of actuator piston position with classical and linear state feedback was described in [14-16]. In this paper, the cascade controller is constructed as a near IO linearizing inner-loop pressure force controller and a feedback plus feed forward outer-loop position controller. The main contribution of the paper is in revealing the equivalence between the cascade from and the near IO linearizing position controller and outlining a physically intuitive gain selection procedure for the latter.

The rest of the paper is organized as follows. Section II presents the system model and lists the relevant assumptions for the controller derivations. In Section III, the near IO linearizing pressure force tracking controller is described. In Section IV, the near IO linearizing position controller and the cascade controller are discussed and their equivalence is revealed. Some experimental results are presented in Section V, and the conclusions in Section VI.

II. SYSTEM MODEL AND BASIC ASSUMPTIONS

A. SYSTEM MODEL

Physical models of electrohydraulic actuators are quite widely available in the literature [16-19]. The model used here applies to a four-way servovalve close-coupled with a piston actuator as shown in Fig. 1. q_t and q_b , are flow rates from the top chamber and to the bottom chamber of the cylinder, respectively. q_i represents internal leakage flow and

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 $q_{e,t}$ and $q_{e,b}$ are external leakage. A_t and A_b represent the effective piston areas, and V_t and V_b designate the volumes of oil in the top and bottom chambers, respectively, corresponding to the center position (x_p =0) of the piston. These include the volumes of oil in the short pipelines between the close-coupled servovalve and actuator.



Fig. 1. Schematic of a rectilinear actuator and servovalve

The chamber pressure dynamics are given by [19]:

$$\frac{dp_b}{dt} = \frac{\beta_e}{V_b + A_b x_p} (q_b - A_b \dot{x}_p + q_i - q_{e,b})$$
(1)

$$\frac{dp_{t}}{dt} = \frac{\beta_{e}}{V_{t} - A_{t}x_{p}} (-q_{t} + A_{t}\dot{x}_{p} - q_{i} - q_{e,t})$$
(2)

The external leakage flows, $q_{e,b}$, and $q_{e,t}$, are negligible. The internal leakage past the piston seals is assumed to be laminar, with a leakage coefficient, C_L :

$$q_i = C_L(p_t - p_b) \tag{3}$$

The (mainly) turbulent flows via the sharp-edged control orifices of the spool valve are modeled by [1, 18, 19]:

$$q_{b} = K_{v,1} sg(x_{v} + u_{1}) sgn(p_{s} - p_{b}) \sqrt{|p_{s} - p_{b}|} - K_{v,2} sg(-x_{v} + u_{2}) sgn(p_{b} - p_{b}) \sqrt{|p_{b} - p_{b}|}$$
(4)

$$q_{t} = K_{v,3} sg(x_{v} + u_{3}) sgn(p_{t} - p_{R}) \sqrt{|p_{t} - p_{R}|} - K_{v,4} sg(-x_{v} + u_{4}) sgn(p_{s} - p_{t}) \sqrt{|p_{s} - p_{t}|}$$
(5)

where the function, sg(x), is defined by:

$$sg(x) = \begin{cases} x, & x \ge 0\\ 0, & x < 0 \end{cases}$$
(6)

Here, $K_{v,i}$, i = 1, 2, 3, 4 are the valve coefficients.

The force on the actuator piston due to the oil pressure is:

$$F_p = A_b p_b - A_t p_t \tag{7}$$

Denoting the friction force on the piston by F_f and the load force (such as a tensile force on fatigue test specimen) by F_L , and applying Newton's Second Law, we have:

$$\dot{x}_p = v_p \tag{8}$$

$$\dot{v}_{p} = \frac{1}{m_{p}} [F_{p} - F_{L} - F_{f} - m_{p}g]$$
(9)

Equations (1), (2), (8) and (9), with q_b and q_t given by (4) and (5), constitute the state space model for the servovalve and loaded actuator system. These equations also contain the major nonlinearities in the system, which are the variable position dependent hydraulic capacitance and the square root flow rate versus pressure drop relations. Further nonlinearity is introduced in (9) by a nonlinear friction force [20] and a possibly nonlinear load force, F_L .

B. Basic Assumptions for Derivation of the Control Laws

The servovalve is assumed to be critically centered, i.e., underlap/overlap lengths are neglected. Instead, an offset value of the valve position is estimated during calibration to take into account abrasion-induced null offsets [19]. Also, the valve spool dynamics are neglected, i.e, the valve spool position is assumed to be related to the servovalve current through a static gain G_{ν} , as:

$$\bar{i}_{\nu} = G_{\nu} \bar{x}_{\nu} \tag{10}$$

where, $\overline{i}_v = i_v - i_{voff}$, and $\overline{x}_v = x_v - x_{voff}$, with i_{voff} and x_{voff} representing the current offset and valve spool position offset, respectively. In this paper, we choose to consider the servovalve current as the control variable. The flow rates to and from the cylinder chambers are then rewritten as:

$$C_{v,1} sg(i_{v}) sgn(p_{s} - p_{b}) \sqrt{|p_{s} - p_{b}|} - C_{v,2} sg(-\bar{i}_{v}) sgn(p_{b} - p_{b}) \sqrt{|p_{b} - p_{b}|}$$

$$(11)$$

$$q_{t} = C_{v,3} sg(\bar{t}_{v}) \operatorname{sgn}(p_{t} - p_{R}) \sqrt{|p_{t} - p_{R}|} -$$
(12)

$$C_{v,4}sg(-\overline{i}_v)sgn(p_s-p_t)\sqrt{|p_s-p_t|}$$

where the new valve coefficients are given by:

$$C_{\nu,i} = G_{\nu} K_{\nu,i}$$
 $i = 1, 2, 3, 4$ (13)

The form of the flow rate equations given by (11) and (12) allow us to estimate the valve coefficients $C_{v,i}$ from quick offline experiments; see for e.g., [1] (pp 184-186). To simplify the analysis, we assume perfect knowledge of the necessary parameters. The supply (p_S) and return (p_R) pressures at the servovalve are also assumed to be constant.

One more assumption is necessary to allow feedback linearization. This regards as to whether the control input appears affine in the system model. This assumption is described following the controller expression (22).

III. PRESSURE FORCE TRACKING CONTROL

Taking the derivative of the pressure force on the piston in (7), and using (1) and (2), it can be shown that:

q

$$\dot{F}_{p} = -\dot{x}_{p}\beta_{e} \left(\frac{A_{b}^{2}}{V_{b} + A_{b}x_{p}} + \frac{A_{t}^{2}}{V_{t} - A_{t}x_{p}} \right) + \frac{A_{b}\beta_{e}}{V_{b} + A_{b}x_{p}} (q_{b} + q_{i}) + \frac{A_{t}\beta_{e}}{V_{t} - A_{t}x_{p}} (q_{t} + q_{i})$$
(14)

Using (11) and (12) for q_b and q_t , (14) can be rewritten as: $\dot{F}_p = f_F(x_p, \dot{x}_p, p_b, p_t) + g_F(x_p, p_b, p_t, \text{sgn}(\bar{i}_v))\bar{i}_v$ (15) where the nonlinear functions, f_F and g_F , are, respectively:

$$f_{F}(x_{p}, \dot{x}_{p}, p_{b}, p_{t}) = -\dot{x}_{p}\beta_{e}\left(\frac{A_{b}^{2}}{V_{b} + A_{b}x_{p}} + \frac{A_{t}^{2}}{V_{t} - A_{t}x_{p}}\right) + (16)$$

$$\frac{A_{b}\beta_{e}C_{L}(p_{t} - p_{b})}{V_{b} + A_{b}x_{p}} + \frac{A_{t}\beta_{e}C_{L}(p_{t} - p_{b})}{V_{t} - A_{t}x_{p}}$$

$$\begin{cases} \frac{A_{b}\beta_{e}C_{v}}{V_{b} + A_{b}x_{p}} \operatorname{sgn}(p_{s} - p_{b})\sqrt{|p_{s} - p_{b}|} \\ + \frac{A_{t}\beta_{e}C_{v}}{V_{v} - A_{t}x_{p}} \operatorname{sgn}(p_{t} - p_{R})\sqrt{|p_{t} - p_{R}|} & \text{for } \bar{i}_{v} \ge 0 \\ \\ \frac{A_{b}\beta_{e}C_{v}}{V_{b} + A_{b}x_{p}} \operatorname{sgn}(p_{b} - p_{R})\sqrt{|p_{b} - p_{R}|} \\ + \frac{A_{t}\beta_{e}C_{v}}{V_{b} + A_{b}x_{p}} \operatorname{sgn}(p_{s} - p_{t})\sqrt{|p_{s} - p_{t}|} & \text{for } \bar{i}_{v} < 0 \\ \end{cases}$$

$$(17)$$

Equation (15), with f_F and g_F defined by (16) and (17), respectively, contains all the major modeled nonlinearities in the hydraulic system arising from fluid compliance and turbulent orifice flow. Also, the derivative of the output piston force, F_p , is seen to be piecewise linear in the control input (\bar{i}_v) . This suggests that piecewise IO linearization can be performed in the domains $(\bar{i}_v \ge 0 \text{ and } \bar{i}_v < 0)[9, 10]$. The nonlinearities in the pressure force dynamics (15) can be cancelled by choosing the control input:

$$\bar{i}_{v} = \frac{1}{g_{F}(x_{p}, p_{b}, p_{t}, \text{sgn}(\bar{i}_{v}))} (v - f_{F}(x_{p}, \dot{x}_{p}, p_{b}, p_{t}))$$
(18)

where v is a new (transformed) control input. The pressure force dynamics (15) reduces to:

$$\dot{F}_p = v \tag{19}$$

This is a simple linear integrator. Exponentially convergent tracking of a desired differentiable piston force profile $(F_{p,d})$ can be achieved by choosing *v* as follows:

$$v = F_{p,d} - k_o (F_p - F_{p,d})$$
(20)

The force tracking error dynamics are given by:

$$e_F + k_o e_F = 0 \tag{21}$$

where e_F is the force tracking error, $e_F = F_p - F_{p,d}$.

In terms of the tracking error, e_F , the control current is:

$$\bar{i}_{v} = \frac{1}{g_{F}(x_{p}, p_{b}, p_{t}, \text{sgn}(\bar{i}_{v}))} (\dot{F}_{p,d} - k_{o}e_{F} - f_{F}(x_{p}, \dot{x}_{p}, p_{b}, p_{t}))$$
(22)

It is important to note that (22) cannot be solved "as is", since it contains the control variable, \bar{i}_v , on both sides of an

equation involving the sgn function. A practical solution to this problem becomes evident when considering the digital implementation of the piecewise IO linearizing controller. The sign of the value of i_v at the previous time step can be used to compute the value of i_v at the current time step, if it can be supposed that the current does not change signs at a rate faster than the sampling rate (approached by using a fast sampling rate). However, it is difficult to analytically prove that this approach does not lead to control chatter. This chatter problem has not been reported previously in the literature that discusses feedback linearization for hydraulic drives [12, 13, 20]. Nevertheless, the term near input-output (near IO) linearization is adopted here to make the explicit admission that the present controller is not a true IO linearizing controller.

The IO linearization thus achieved is of relative degree one. The stability of the internal dynamics rendered 'unobservable' is easily handled by introducing the concept of the load pressure/differential pressure [1, 18, 20].

IV. PISTON POSITION TRACKING CONTROL

A. Near IO linearizing Position Controller

In this subsection, piecewise IO linearization is performed with piston position, x_p , as the system output. The first and second derivatives of the output position, x_p , as given by (8) and (9) do not contain the control input, \bar{i}_v , However, further differentiation of (9) gives:

$$\ddot{x}_{p} = f_{p}(x_{p}, \dot{x}_{p}, p_{b}, p_{t}, \dot{F}_{f}, \dot{F}_{L}) + g_{p}(x_{p}, p_{b}, p_{t}, \text{sgn}(\bar{i}_{v}))\bar{i}_{v}$$
(23)

where the nonlinear functions, f_p and g_p , are respectively:

$$f_{p}(x_{p}, \dot{x}_{p}, p_{b}, p_{t}, \dot{F}_{f}, \dot{F}_{L}) = \frac{1}{m_{p}} [f_{F}(x_{p}, \dot{x}_{p}, p_{b}, p_{t}) - F_{f} - F_{L}]$$
(24)

$$g_{p}(x_{p}, p_{b}, p_{t}, \text{sgn}(\bar{i}_{v})) = \frac{1}{m_{p}} g_{F}(x_{p}, p_{b}, p_{t}, \text{sgn}(\bar{i}_{v}))$$
 (25)

Proceeding as in the last subsection, (23) leads to a piecewise IO linearization suggesting the control law:

$$\bar{i}_{v} = \frac{1}{g_{p}(x_{p}, p_{b}, p_{t}, \text{sgn}(\bar{i}_{v}))} (v - f_{p}(x_{p}, \dot{x}_{p}, p_{b}, p_{t}, \dot{F}_{f}, \dot{F}_{L}))$$
(26)

The closed loop position dynamics reduces to:

$$\ddot{x}_p = v$$
 (27)

It leads to an exponentially convergent tracking when the new input v is chosen as:

$$v = \ddot{x}_d - k_3(\ddot{x}_p - \ddot{x}_d) - k_2(\dot{x}_p - \dot{x}_d) - k_1(x_p - x_d)$$
(28)

where x_d is the desired position profile. The dynamics of the closed loop position tracking error, $e=x_p-x_d$, reduce to:

$$\ddot{e} + k_3 \ddot{e} + k_2 \dot{e} + k_1 e = 0$$
 (29)

The control law is rewritten as:

$$\bar{i}_{v} = \frac{1}{g_{p}(x_{p}, p_{b}, p_{t}, \text{sgn}(\bar{i}_{v}))} (\ddot{x}_{d} - k_{3}\ddot{e} - k_{2}\dot{e} - k_{1}e - f_{p}(x_{p}, \dot{x}_{p}, p_{b}, p_{t}, \dot{F}_{f}, \dot{F}_{L}))$$
(30)

The three gains k_1 , k_2 , and k_3 can be chosen to place the poles of the closed loop tracking error dynamics (29) strictly in the left half s-plane. This could be done by using direct pole placement or posing the problem as a linear optimal control (such as LQR) problem [20]. However, neither approach offers a clear physically intuitive guide for the choice of the gains. In the next subsections, another approach is revealed.

It should be noted again that the piecewise IO linearization performed is only a near IO linearization with the description of the internal dynamics better handled using the load pressure description [1, 18, 20].

B. Cascade Position Control

Recall that the near IO linearizing pressure force controller cancels the natural velocity feedback on the pressure force dynamics (15) by the first term of the nonlinear function, f_F . This cancellation decouples the dynamics of the piston motion from the hydraulic pressure/force dynamics. This fact forms the basis for the cascade control of piston position by allowing one to treat the actuator as a force generator with an inner-loop force F_p tracking controller, and a feedback plus feed forward outer-loop position controller computing the desired force profile.

Heintze and Van der Welden [14] compared an inner-loop controller based on dynamic inversion with a cascade controller which includes nonlinearity compensation in the original constant-gain cascade form of Sepehri, et al [15]. Starting with a Lyapunov-like analysis, Sohl and Bobrow [8] also presented a cascade position tracking controller. In this paper, we focus on revealing certain useful facts about the nonlinear cascade controller from a feedback linearization framework.

We construct the desired pressure force profile $(F_{p,d})$ in terms of the desired piston position profile in such a manner that when the pressure force output is driven to the desired force profile, the piston position output approaches the desired position. Define $F_{p,d}$ as:

$$F_{p,d} = m_p \ddot{x}_d - k_v (\dot{x}_p - \dot{x}_d) - k_p (x_p - x_d) + F_L + F_f + m_p g$$
(31)

It is assumed here that accurate estimates of the friction force and the load force are available and the piston mass is known. The choice of the gains of k_v and k_p will be discussed shortly. To further appreciate the choice of the form of $F_{p,d}$ in (31), recall the equation of motion:

$$m_p \ddot{x}_p = F_p - F_L - F_f - m_p g \tag{32}$$

Combining (31) and (32), the closed loop dynamics can be expressed in terms of the position error, $e=x_p-x_d$.

$$m_{p}\ddot{e} + k_{v}\dot{e} + k_{p}e = F_{p} - F_{p,d} = e_{F}$$
(33)

where $e_F = F_p - F_{p,d}$ is the pressure force tracking error. It has already been argued that e_F can be driven to zero using the controller given by (22). Equation (33) shows that the position error dynamics are given by a second-order linear differential equation driven by e_F . The gains k_v and k_p can then be chosen to obtain a desired position error dynamics. Fig. 2 shows a schematic for the implementation of this cascade control structure.



Fig. 2. Schematic of the cascade controller (known friction and load)

C. Equivalence of the two position controllers

First, note that the closed loop system with the cascade position controller is of order three, as is the one with the near IO linearizing position controller. Taking the derivative of the desired pressure force (31), and using the result together with (33) in the near IO linearizing pressure force tracking controller of (22) it can be shown that:

$$\bar{i}_{v} = \frac{\ddot{x}_{d} - (k_{o} + \frac{k_{v}}{m_{p}})\ddot{e} - \frac{(k_{p} + k_{o}k_{v})}{m_{p}}\dot{e} - \frac{k_{o}k_{p}}{m_{p}}e - \frac{(f_{F} - \dot{F}_{f} - \dot{F}_{L})}{m_{p}}}{\frac{g_{F}}{m_{p}}}$$
(34)

where f_F and g_F are given by (16) and (17), respectively. Note that (34) has the same form as the near IO linearizing position tracking controller given by (30). The two controllers will be exactly the same when the gains satisfy:

$$k_1 = \frac{k_o k_p}{m_p}, \ k_2 = \frac{k_p + k_o k_v}{m_p}, \ k_3 = k_o + \frac{k_v}{m_p}$$
 (35)

This shows the equivalence of the two position controllers. Note that both the cascade controller leading to (34) and the near IO linearizing position controller (30) have three linear gains to be set. The question to pose at this point may be: Which controller structure is better?

The cascade controller has certain apparent advantages over the near IO linearizing controller. First, it gives a simpler physical insight and interpretation that can aid in the choice of the linear gains. The inner-loop pressure force dynamics can be made as fast as desired by the choice of k_o via pole placement ($s=-k_o$) of a first order linear dynamics (21). The other gains, k_p and k_v , are simply coefficients of a second-order linear dynamics (33) and they are easily interpreted as factors in the natural frequency and damping coefficient of the position outer-loop. A second advantage is that the cascade form does not need feedback of acceleration measurement (see (31)). It also does not require a third derivative of the desired position trajectory to be available, even though it is implicit that the third derivative must be bounded due to the fact that the derivative of $F_{p,d}$ is used by the inner-loop force controller.

However, the cascade control structure has a serious disadvantage in that it involves online differentiation of the desired force, $F_{p,d}$, which in itself is computed online from feedback of position, velocity, and pressure, as well as the estimates of friction and load forces. Since these signals are susceptible to noise, high quality signal processing may be necessary. The near IO linearizing position tracking controller does not do such online differentiations.

Note that the gain relations (35) can be used in a reverse argument (solving for k_1 , k_2 , k_3) to guide the choice of the gains for the near IO linearizing controller of (30). However, closed form inversion of (35) results in complicated and unwieldy expressions for the general case, and as such readily available numerical solutions are recommended.

For the special case where all three closed-loop poles of (29) are placed at the same location on the real axis, say s=-a, $a \in \mathcal{R}^+$, a simple, yet, very useful closed form result can be derived. By expanding the characteristic polynomial $(s+a)^3$ of (29) and using the equivalence in (35) one can arrive at:

$$k_o = a, \quad k_p = a^2 m_p, \quad k_v = 2am_p$$
 (36)

In fact, with the result in (36), the second-order position error dynamics has a natural frequency of a (rad/s) and a damping ratio of 1. A critically damped response is among well behaved responses often chosen for the design of position tracking outer-loop. Since $k_o = a$, the first-order force tracking inner-loop also has a break frequency of a.

The observation above provides a powerful tool for the design of the cascade controller and also the near IO linearizing position controller. This is particularly significant for the latter since the interpretation of the gains k_1 , k_2 , k_3 , of the near IO linearizing controller is not readily evident in the third order dynamics of (29), but these gains can be tuned indirectly by first specifying k_o , k_v , and k_p of the cascade interpretation and using the equivalence (35).

V. EXPERIMENTAL RESULTS

To demonstrate the use of the equivalence identified above, some experimental results are included in this section. We first remark that the test system employed does not satisfy some of the assumptions made for controller derivation in Section II. In particular, the servovalve dynamics was not necessarily negligible, and the supply and return pressure were not necessarily constant because of 3-m long lines between the accumulators and the servovalve ports. The results presented here are intended to mainly demonstrate how the equivalence derived above can be used to tune the nonlinear position controllers (the cascade form or the near IO linearizing controller). In these tests, the load force F_L was zero and the friction F_f was estimated in real time using classical approximations of static (Stribeck) plus Coulomb plus viscous [20] curves identified from offline experiments. For the tests, a smooth step reference was created using the tangent hyperbolic function.



Fig.3. Equivalence of the near IO linearizing controller and the cascade form. All poles chosen at s=-250.

Fig. 3 shows an experimental confirmation of the equivalence of the near IO linearizing position controller and the cascade controller. The three closed loop poles of (29) were placed at *s*=-250. The resulting numerical values of the cascade gains computed by (36) are k_p =6.95E5 kg s⁻², k_v =5560 kg s⁻¹, and k_o =250 s⁻¹ for m_p =11.12 kg. Note that nearly identical performance was obtained despite the differences in the two controller structures. However, it remains possible that with different feedback signal processing provisions, any of the strengths and weaknesses of the two controllers noted above could surface.



Fig. 4 Tuning the performance of the nonlinear position controllers with the k_p gain starting with all poles at s=-202.

Fig. 4 shows experimental results showing tuning of the position controller (either the near IO linearizing or the cascade controller) by changing the gain k_o of the force loop independently of the outer position loop. k_p and k_v were kept constant. The tuning was started with all pole locations at s=-202 which corresponds to k_p =4.45E5 kg/s², k_v =4490 kg/s for m_p =11.12 kg, i.e, the second-order position error dynamics has a natural frequency of 32 Hz and critical damping, and

the force loop has a break frequency of 32 Hz.

Fig. 5 demonstrates tuning with the position loop gain k_p of the cascade. Similar tuning can be done using k_v .



Fig. 5. Tuning with the gain k_p starting with all poles at s=-202.

VI. CONCLUSIONS

In this paper, two nonlinear controller structures were outlined for the position control of an electrohydraulic actuator. The first was a near IO linearization of the model of the actuator with position output. And the second was a cascade controller, comprising of a near IO linearizing pressure force controller as an inner-loop to a feedback plus feed forward position controller as outer-loop.

It was shown that the two controllers are equivalent, save for their possible differences due to signal processing upon implementation. Expressions were derived for the interrelationship between the gains of the two controllers and a simple procedure for tuning either controller was outlined:

- 1. Choose the gains of the cascade controller: start with all three poles of (29) at s=-a ($a \in \mathfrak{R}^+$), and use (36).
- 2. Tune the performance with k_o , k_p , and/or k_v for the cascade, or equivalently k_1 , k_2 , k_3 obtained by inverting (35) for the near IO linearizing controller.

This procedure is particularly significant for tuning the linear gains k_1 , k_2 , k_3 of the near IO linearizing position controller, for which simple pole placement and optimal criteria (such as LQR) do not yield physically intuitive tuning rules.

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